ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume X (XX), 2011, NR1

THE STUDY OF THE BEHAVIOUR IN SERVICE OF A HYDRAULIC DUPLICATING ATTACHMENT

Adina Cataneanu, Dan Gheorghe Bagnaru, Mihnea Cataneanu
University of Craiova, Faculty of Mechanics, adina_cataneanu@yahoo.com

Keywords: vibrations**,** hydraulic duplicating attachment, resonance

Abstract. The mechanical model of a hydraulic duplicating attachment is considered. Its valve is moving on the vertical direction droved by one roll through the agency of a spring. The piece which is subjected to the duplicate work has a sinusoidal profile and moves together with the live unit of the producer on which is mounted. The motion law of the valve is determined in two cases: one on the assumption that the dying-out exists and the other considering the dying-out negligible. The critical rate of travel of the piece is determined. This corresponds to the resonance phenomenon which provokes the fracture of the pieces submitted to the vibrations.

1. THE MECHANICAL MODEL OF MOTION

In the figure 1, the mechanical model of a hydraulic duplicating attachment is presented.

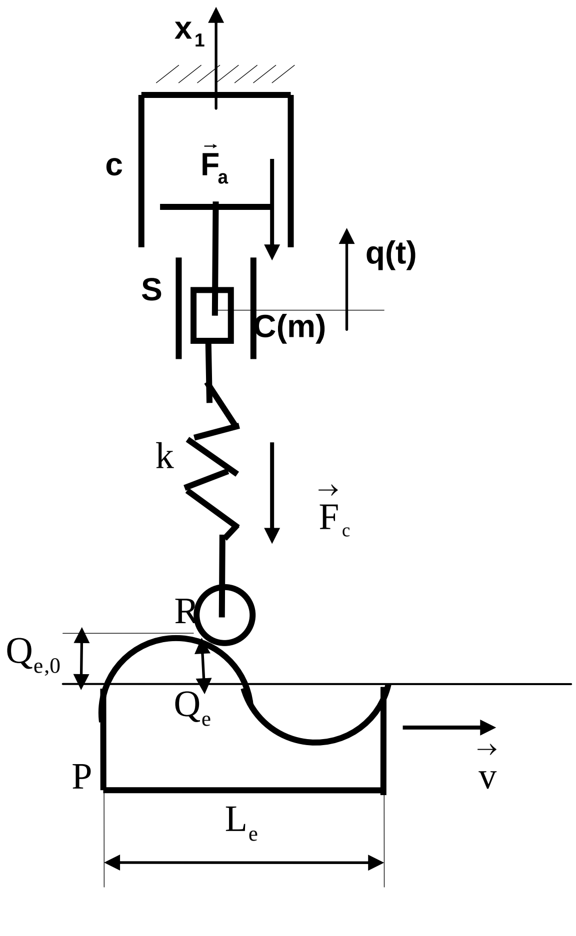

Fig. 1 The mechanical model of the hydraulic duplicating attachment

The valve S of the duplicating attachment is moving on a vertical direction droved by the roll R, through the agency of a spring with the spring rate K. The piece P, which is subjected to the duplicate work, named template follower, has a sinusoidal profile, given by the function:

$$Q_e = Q_{e,0} \sin(\omega_e t), \tag{1}$$

with its period L_e.

Also, the piece P has a rate of travel $\vec{v}$, performed by the live unit of the producer on which it is mounted.

2. THE MATHEMATICAL MODEL OF MOTION

The present study intends to determine the motion equation of the valve S in two cases: one supposing that the damping exists and the other considering the damping negligible. Further on, the motion law of the valve is determined in both cases.

As things are, the gravitational force $\vec{G}$ is not present in the movement differential equation. The displacement $\vec{Q}_e$ plays the part of an external disturbance. The mass m of the valve S is considered to be concentrated in its mass center C.

Applying the fundamental principle of dynamics, the equation of the motion can be written:

$$m\vec{a} = \vec{F}_a + \vec{F}_e, \tag{2}$$

where:

$$\vec{F}_a = -c\dot{q}\,\vec{i}_1, \quad .\vec{F}_e = -k(q - Q_e)\vec{i}_1, \quad \vec{a} = \ddot{q}\,\vec{i}_1$$

On the Ox_1 axis projection, it results:

$$m\ddot{q} = -c\dot{q} - kq + kQ_{e,0}\sin(\omega_e t). \tag{3}$$

It must be observe that the fixed point O represents the static equilibrium position. Dividing by m, in the relation (3), it is obtained:

$$\ddot{q} + 2n\dot{q} + \omega_n^2 q = f_0 \sin(\omega_e t), \tag{3'}$$

where:

$$2n = \frac{c}{m}; \tag{4}$$

$$\omega_n^2 = \frac{k}{m}: \tag{5}$$

$$f_0 = \frac{kQ_{e,0}}{m} = \omega_n^2 Q_{e,0}. \tag{6}$$

Considering the dying-out negligible, with $\vec{F}_a = \vec{0}$, the motion mathematical model becomes:

$$\ddot{q} + \omega_n^2 = f_0 \sin(\omega_e t). \tag{3''}$$

3. THE MOTION LAW OF THE VALVE

By applying the Laplace transform in relation to time in the equation (3”), the following algebraic equation is obtained:

$$s^2\tilde{q}(s) - sq(0) - \dot{q}(0) + \omega_n^2\tilde{q}(s) = \frac{f_0\omega_e}{s^2 + \omega_e^2}, \tag{7}$$

or, with $q(0) = q_0$, $\dot{q}(0) = v_0$, the equation becomes:

$$s^2\tilde{q}(s) - sq_0 - v_0 + \omega_n^2\tilde{q}(s) = \frac{f_0\omega_e}{s^2 + \omega_e^2}, \tag{7'}$$

with the solution:

$$\tilde{q}(s) = \frac{sq_0 + v_0}{s^2 + \omega_n^2} + \frac{f_0\omega_e}{\left(s^2 + \omega_n^2\right)\left(s^2 + \omega_e^2\right)}, \tag{8}$$

or, in different manner writing:

$$\tilde{q}(s) = \frac{v_0}{\omega_n} \cdot \frac{\omega_n}{s^2 + \omega_n^2} + q_0 \cdot \frac{s}{s^2 + \omega_n^2} + \frac{f_0\omega_e}{\left(s^2 + \omega_n^2\right)\left(s^2 + \omega_e^2\right)}. \tag{8'}$$

If, in the equation (8'), the inverse of the Laplace transform is applied, it results the time function:

$$q(t) = \frac{v_0}{\omega_n}\sin(\omega_n t) + q_0\cos(\omega_n t) + \frac{f_0\omega_e}{\omega_n\left(\omega_e^2 - \omega_n^2\right)}\sin(\omega_n t) + \frac{f_0}{\omega_n^2 - \omega_e^2}\sin(\omega_e t). \tag{9}$$

Using the expression (6) and, conveniently, grouping the terms, the relation (9) becomes:

$$q(t) = \left[\left(\frac{v_0}{\omega_n} + \frac{\omega_n\omega_e Q_{e,0}}{\omega_e^2 - \omega_n^2}\right)\sin(\omega_n t) + q_0\cos(\omega_n t)\right] + \frac{\omega_n^2 Q_{e,0}}{\omega_n^2 - \omega_e^2}\sin(\omega_e t) = q_n + q_e, \tag{10}$$

where:

$$q_n(t) = \left(\frac{v_0}{\omega_n} + \frac{\omega_n\omega_e Q_{e,0}}{\omega_e^2 - \omega_n^2}\right)\sin(\omega_n t) + q_0\cos(\omega_n t); \tag{10'}$$

$$q_e(t) = \frac{\omega_n^2 Q_{e,0}}{\omega_n^2 - \omega_e^2}\sin(\omega_e t). \tag{10''}$$

The function $q(t)$, given by the relation (10), is represented in the figure 2 and results through summation of two parallel vibrations with distinct pulsations, namely the natural vibration $q_n(t)$ and the undamped vibration $q_e(t)$. The first term $q_n(t)$ characterizes the vibrations from the insertion process.

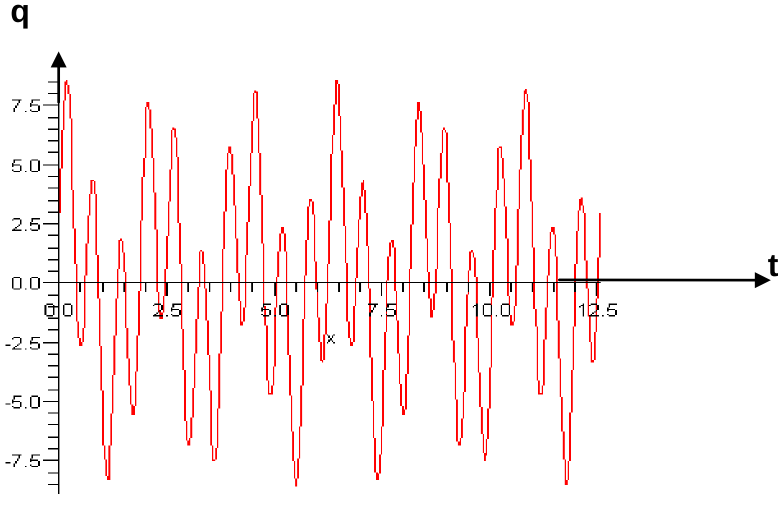

Fig. 2 The representation of the function q(t)

The study of the undamped vibrations, representing the stationary regime for the action of the disturbance $\vec{Q}_e$ on the same elastic system, is done taking into account only the term $q_e(t)$ given by the relation (10”). This term can be written:

$$q_e(t) = Q_{e,0} \frac{1}{1-\left(\frac{\omega_e}{\omega_n}\right)^2} \sin(\omega_e t) = A_e \sin(\omega_e t), \qquad (10”’)$$

where the amplitude of the undamped vibration is:

$$A_e = Q_{e,0} \frac{1}{1-\left(\frac{\omega_e}{\omega_n}\right)^2} Q_{e,0} \cdot A, \qquad (11)$$

$$A = A\left(\frac{\omega_e}{\omega_n}\right) = \frac{1}{\left|1-\left(\frac{\omega_e}{\omega_n}\right)^2\right|} \qquad (11’)$$

representing the multiplicator factor. By means of the multiplicator factor, a qualitative study of the motion can be do, like in figure 3. The ratio $\frac{\omega_e}{\omega_n}$ is named the relative pulsation. The appearance of the modulus in the relation (11’) is legitimate by the fact that the two pulsations, the natural pulsation ω_n and the undamped pulsation ω_e, are very distinct, that is $\omega_n << \omega_e$ or $\omega_n >> \omega_e$. The absolute value of the dynamical deformation is very important. There are more interesting zones in the graphical representation from figure 3.

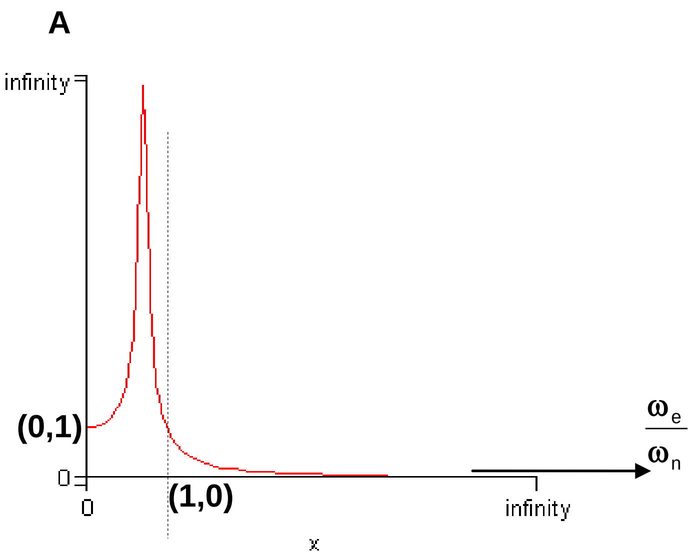

Fig.3 The representation of the multiplicator factor in relation to the relative pulsation

1. The point (0,1) corresponds to the value $\frac{\omega_e}{\omega_n} = 0$, thus $A = 1$ and $A_e = Q_{e,0}$, that is the system vibrates in phase with the excitation.

2. If $\omega_n << \omega_e$ that is $\frac{\omega_e}{\omega_n} \to \infty$, it must to observe that $A \to 0 \Rightarrow A_e \to 0$. In conclusion, we can say that the influence of the disturbance is, virtually, null.

3. The diagram has a vertical asymptote $\frac{\omega_e}{\omega_n} = 1$, that is:

$$\lim_{\frac{\omega_e}{\omega_n} \to 1} A = \lim_{\frac{\omega_e}{\omega_n} \to 1} \frac{1}{\left| 1 - \left(\frac{\omega_e}{\omega_n} \right)^2 \right|} = +\infty \tag{12}$$

and, in accordance with the relation (11), the dynamical amplitude $A_e \to \infty$. This way, the resonance phenomenon appears and provokes the fracture of the pieces submitted to the vibration. In this view, it is necessary to avoid that the two pulsations been equal.

It is possible to come to the same results if the integration of the differential equation (3") is resumed with $\omega_e = \omega_n$. Then, the equation gets the form:

$$\ddot{q} + \omega_n^2 = f_0 \sin(\omega_n t). \tag{3"'}$$

Applying to the equation (3"') the Laplace transform in relation to time, it results the algebraic equation:

$$s^2 \tilde{q}(s) - s q(0) - \dot{q}(0) + \omega_n^2 \tilde{q}(s) = \frac{f_0 \omega_n}{s^2 + \omega_n^2}, \tag{13}$$

or, with $q(0) = q_0$, $\dot{q}(0) = v_0$, the equation becomes:

$$s^2 \tilde{q}(s) - s q_0 - v_0 + \omega_n^2 \tilde{q}(s) = \frac{f_0 \omega_n}{s^2 + \omega_n^2}, \tag{13'}$$

with the solution:

$$\tilde{q}(s) = \frac{s q_0 + v_0}{s^2 + \omega_n^2} + \frac{f_0 \omega_n}{\left(s^2 + \omega_n^2\right)^2}, \tag{14}$$

or, in different manner writing:

$$\tilde{q}(s) = \frac{v_0}{\omega_n} \cdot \frac{\omega_n}{s^2 + \omega_n^2} + q_0 \cdot \frac{s}{s^2 + \omega_n^2} + \frac{f_0 \omega_n}{\left(s^2 + \omega_n^2\right)^2}. \tag{14'}$$

By inverting this relation, it results the solution:

$$q(t) = \frac{v_0}{\omega_n} \sin(\omega_n t) + q_0 \cos(\omega_n t) + \frac{f_0}{2\omega_n^2} \sin(\omega_n t) - \frac{f_0}{2\omega_n} t \cos(\omega_n t). \tag{15}$$

Using the expression (6) and, conveniently, grouping the terms, the solution (15) becomes:

$$q(t)=\left[\left(\frac{v_0}{\omega_n}+\frac{f_0}{2\omega_n^2}\right)\sin(\omega_n t)+q_0\cos(\omega_n t)\right]-\frac{\omega_n Q_{e,0}}{2}t\cos(\omega_e t)=q_n+q_e, \tag{15'}$$

where:

$$q_n(t)=\left(\frac{v_0}{\omega_n}+\frac{f_0}{2\omega_n^2}\right)\sin(\omega_n t)+q_0\cos(\omega_n t); \tag{15''}$$

$$q_e=-\frac{\omega_n Q_{e,0}}{2}t\cos(\omega_e t) \tag{15'''}$$

The undamped vibration given by (15''') has its amplitude the function time:

$$A_e(t)=-\frac{\omega_n Q_{e,0}}{2}t. \tag{16}$$

Such vibration is named quasi-harmonic with amplitude modulation and is represented in the figure 4. It must observe that the motion given by the law (15''') is unstable and its amplitude has an infinite increase, as is shown in figure 4.

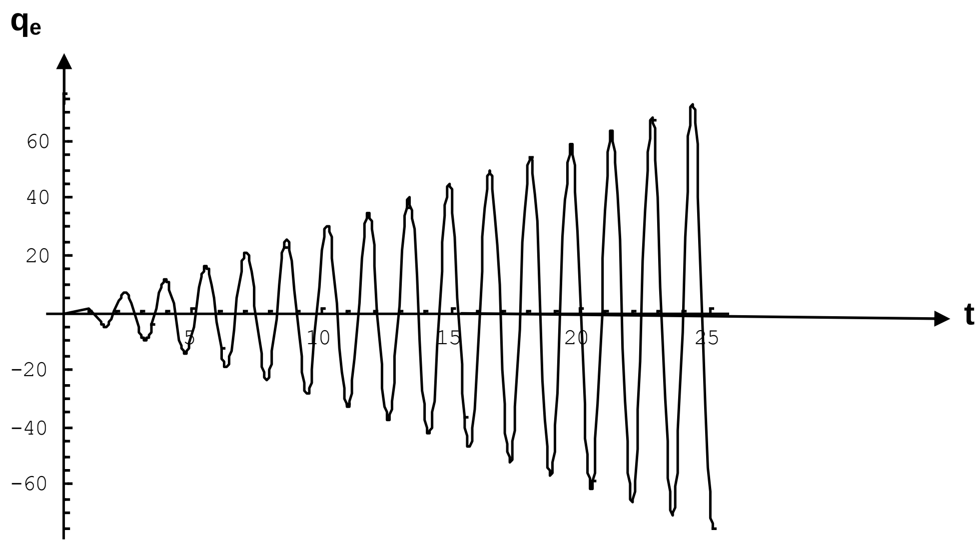

Fig.4 The representation of the function $q_e(t)$

5. DETERMINATION OF THE CRITICAL RATE OF TRAVEL OF THE PIECE

The period of the undamping vibrating movement is:

$$T_e=\frac{2\pi}{\omega_e}. \tag{17}$$

During one period, the piece advances with the length:

$$L_e = v \cdot T_e. \tag{18}$$

From the relations (17) and (18) results the pulsation of the undamping vibration:

$$\omega_e = \frac{2\pi v}{L_e}. \tag{19}$$

The resonance takes place when $\omega_e = \omega_n$. So, from the relations (4) and (19), the critical rate of travel of the piece, which is subjected to the duplicate, is obtained:

$$v_{cr} = \frac{L_e}{2\pi} \cdot \sqrt{\frac{k}{m}}. \tag{20}$$

6. CONCLUSIONS

The study of the mechanical model of the hydraulic duplicating attachment leads, finally, to obtain the motion law of the valve in both cases: one supposing that the damping exists and the other considering the damping negligible.

One can easily notice that, for a good working of the device, it is necessary to avoid the resonance phenomenon that causes fractures of the pieces submitted to vibrations. In this view, the critical rate of travel of the piece, which is subjected to the duplicate, was obtained and must to be taken into consideration.

Bibliography

[1] Băgnaru, D. Gh., Vibraţiile elementelor cinematice, Editura SITECH, ISBN 975-657-854-2, Craiova, 2005.
[2] Băgnaru, D. Gh., About the vibrations of vehicles, Analele Universităţii din Craiova, Seria Mecanică, Nr. 1, 2004, pag. 7-14, ISSN 1223-5296.
[3] Băgnaru, D. Gh., Cătăneanu, A., About the Determined Vibrations of the Vehicles XVI International Conference on MATERIAL FLOW, MACHINES AND DEVICES IN INDUSTRY", University of Belgrade, Dec.7-8, 2000, pag.16-20.
[4] Băgnaru, D. Gh., Cătăneanu, A., Mecanică – Mecanisme, Editura SITECH, ISBN 973-97524-12-8, Craiova, 1998.
[5] Băgnaru, D. Gh., Cătăneanu, A., Dinamică cu aplicaţii în inginerie, Editura Universitaria, ISBN 978-606-510-574-4, Craiova, 2009
[6] Cătăneanu, A., Băgnaru, D. Gh., Mecanica mediului continuu, Ed. SITECH, ISBN 973-8025-68-0, Craiova, 2000.